# <span id="page-0-0"></span>Package: geostan (via r-universe)

August 20, 2024

Title Bayesian Spatial Analysis

Version 0.6.2

Date 2024-05-31

URL <https://connordonegan.github.io/geostan/>

BugReports <https://github.com/ConnorDonegan/geostan/issues>

Description For spatial data analysis; provides exploratory spatial analysis tools, spatial regression models, disease mapping models, model diagnostics, and special methods for inference with small area survey data (e.g., the America Community Survey (ACS)) and censored population health surveillance data. Models are pre-specified using the Stan programming language, a platform for Bayesian inference using Markov chain Monte Carlo (MCMC). References: Carpenter et al. (2017) [<doi:10.18637/jss.v076.i01>](https://doi.org/10.18637/jss.v076.i01); Donegan (2021) [<doi:10.31219/osf.io/3ey65>](https://doi.org/10.31219/osf.io/3ey65); Donegan (2022)  $\langle \text{doi:10.21105/} \rangle$ joss.04716>; Donegan, Chun and Hughes (2020) [<doi:10.1016/j.spasta.2020.100450>](https://doi.org/10.1016/j.spasta.2020.100450); Donegan, Chun and Griffith (2021) [<doi:10.3390/ijerph18136856>](https://doi.org/10.3390/ijerph18136856); Morris et al. (2019) [<doi:10.1016/j.sste.2019.100301>](https://doi.org/10.1016/j.sste.2019.100301).

License GPL  $(>= 3)$ 

Encoding UTF-8

LazyData true

Roxygen list(markdown = TRUE)

RoxygenNote 7.3.1

**Biarch** true

**Depends**  $R$  ( $>= 3.4$ )

**Imports** spdep ( $>= 1.0$ ), sf ( $>= 1.0-10$ ), ggplot2 ( $>= 3.0.0$ ), methods, graphics, stats, MASS, truncnorm, signs, gridExtra, utils, Matrix (>= 1.3), Rcpp (>= 0.12.0), RcppParallel (>= 5.0.1), rstan ( $> = 2.26.0$ ), rstantools ( $>= 2.1.1$ )

<span id="page-1-0"></span>2 geostan-package

**LinkingTo** BH ( $>= 1.66.0$ ), Rcpp ( $>= 0.12.0$ ), RcppEigen ( $>= 0.3.3.3.0$ ), RcppParallel ( $>= 5.0.1$ ), rstan ( $>= 2.26.0$ ), StanHeaders ( $>=$ 2.26.0)

Suggests testthat, knitr, rmarkdown, bayesplot

SystemRequirements GNU make

VignetteBuilder knitr

Repository https://connordonegan.r-universe.dev

RemoteUrl https://github.com/connordonegan/geostan

RemoteRef HEAD

RemoteSha c330dfcf72a1b5bd0d2371aa75c81d72cbddaec8

# **Contents**



geostan-package *The geostan R package.*

# Description

Bayesian spatial modeling powered by Stan. geostan provides access to a variety of hierarchical spatial models using the R formula interface, supporting a complete spatial analysis workflow with a suite of spatial analysis tools. It is designed primarily for public health and social science research but is generally applicable to modeling areal data. Unique features of the package include its spatial measurement error model (for inference with small area estimates such as those from the American Community Survey), its fast proper conditional autoregressive (CAR) and simultaneous autoregressive (SAR) models, and its eigenvector spatial filtering (ESF) models. The package also supports spatial regression with raster layers.

# geostan-package 3

#### Author(s)

Maintainer: Connor Donegan <connor.donegan@gmail.com> [\(ORCID\)](https://orcid.org/0000-0002-9698-5443)

Other contributors:

- Mitzi Morris [contributor]
- Amy Tims [contributor]

#### References

Carpenter, B., Gelman, A., Hoffman, M.D., Lee, D., Goodrich, B., Betancourt, M., Brubaker, M., Guo, J., Li, P., Riddell, A., 2017. Stan: A probabilistic programming language. Journal of statistical software 76. [doi:10.18637/jss.v076.i01.](https://doi.org/10.18637/jss.v076.i01)

Donegan, C., Y. Chun and A. E. Hughes (2020). Bayesian estimation of spatial filters with Moran's Eigenvectors and hierarchical shrinkage priors. *Spatial Statistics*. [doi:10.1016/j.spasta.2020.100450](https://doi.org/10.1016/j.spasta.2020.100450) (open access: [doi:10.31219/osf.io/fah3z\)](https://doi.org/10.31219/osf.io/fah3z).

Donegan, Connor and Chun, Yongwan and Griffith, Daniel A. (2021). Modeling community health with areal data: Bayesian inference with survey standard errors and spatial structure. *Int. J. Env. Res. and Public Health* 18 (13): 6856. [doi:10.3390/ijerph18136856.](https://doi.org/10.3390/ijerph18136856) Supplementary material: <https://github.com/ConnorDonegan/survey-HBM>.

Donegan, Connor (2021). Building spatial conditional autoregressive models in the Stan programming language. *OSF Preprints*. [doi:10.31219/osf.io/3ey65.](https://doi.org/10.31219/osf.io/3ey65)

Donegan, Connor (2022) geostan: An R package for Bayesian spatial analysis. *The Journal of Open Source Software*. 7, no. 79: 4716. [doi:10.21105/joss.04716.](https://doi.org/10.21105/joss.04716)

Gabry, J., Goodrich, B. and Lysy, M. (2020). rstantools: Tools for developers of R packages interfacing with Stan. R package version 2.1.1 <https://mc-stan.org/rstantools/>.

Morris, M., Wheeler-Martin, K., Simpson, D., Mooney, S. J., Gelman, A., & DiMaggio, C. (2019). Bayesian hierarchical spatial models: Implementing the Besag York Mollié model in stan. Spatial and spatio-temporal epidemiology, 31, 100301. [doi:10.1016/j.sste.2019.100301.](https://doi.org/10.1016/j.sste.2019.100301)

Stan Development Team (2019). RStan: the R interface to Stan. R package version 2.19.2. [https:](https://mc-stan.org) [//mc-stan.org](https://mc-stan.org)

#### See Also

Useful links:

- <https://connordonegan.github.io/geostan/>
- Report bugs at <https://github.com/ConnorDonegan/geostan/issues>

<span id="page-3-1"></span><span id="page-3-0"></span>

Creates a list of connected nodes following the graph representation of a spatial connectivity matrix.

#### Usage

edges(C, unique\_pairs\_only = TRUE, shape)

#### Arguments



#### Details

This is used internally for [stan\\_icar](#page-19-1), can be helpful for creating the scaling factor for BYM2 models fit with [stan\\_icar](#page-19-1), and can be used for visualizing a spatial connectivity matrix.

# Value

If shape is missing, this returns a data. frame with three columns. The first two columns (node1 and node2) contain the indices of connected pairs of nodes; only unique pairs of nodes are included (unless unique\_pairs\_only = FALSE). The third column (weight) contains the corresponding matrix element, C[node1, node2].

If shape is provided, the results are joined to an sf object so the connections can be visualized.

#### See Also

[shape2mat](#page-11-1), [prep\\_icar\\_data](#page-0-0), [stan\\_icar](#page-19-1)

```
data(sentencing)
C <- shape2mat(sentencing)
nbs <- edges(C)
head(nbs)
## similar to:
head(Matrix::summary(C))
head(Matrix::summary(shape2mat(georgia, "W")))
```
# <span id="page-4-0"></span>expected\_mc 5

```
## add geometry for plotting
library(sf)
E \leq - edges(C, shape = sentencing)
g1 = st\_geometry(E)g2 = st_geometry(sentencing)
plot(g1, lwd = .2)plot(g2, add = TRUE)
```
expected\_mc *Expected value of the residual Moran coefficient*

# Description

Expected value for the Moran coefficient of model residuals under the null hypothesis of no spatial autocorrelation.

# Usage

expected\_mc(X, C)

#### Arguments



#### Value

Returns a numeric value.

#### Source

Chun, Yongwan and Griffith, Daniel A. (2013). Spatial statistics and geostatistics. Sage, p. 18.

```
data(georgia)
C <- shape2mat(georgia)
X <- model.matrix(~ ICE + college, georgia)
expected_mc(X, C)
```
<span id="page-5-1"></span><span id="page-5-0"></span>

Extract eigenfunctions of a connectivity matrix for spatial filtering

#### Usage

make\_EV(C, nsa = FALSE, threshold =  $0.2$ , values = FALSE)

# Arguments



# Details

Returns a set of eigenvectors related to the Moran coefficient (MC), limited to those eigenvectors with  $|MC|$  > threshold if nsa = TRUE or  $MC$  > threshold if nsa = FALSE, optionally with corresponding eigenvalues.

# Value

A data.frame of eigenvectors for spatial filtering. If values=TRUE then a named list is returned with elements eigenvectors and eigenvalues.

# Source

Daniel Griffith and Yongwan Chun. 2014. "Spatial Autocorrelation and Spatial Filtering." in M. M. Fischer and P. Nijkamp (eds.), *Handbook of Regional Science.* Springer.

# See Also

[stan\\_esf,](#page-13-1) [mc](#page-6-1)

<span id="page-6-0"></span> $m$   $\sim$  7

# Examples

```
library(ggplot2)
data(georgia)
C <- shape2mat(georgia, style = "B")
EV < - make_EV(C)head(EV)
ggplot(georgia) +
  geom_sf(aes(fill = EV[, 1])) +scale_fill_gradient2()
```
<span id="page-6-1"></span>

#### mc *The Moran coefficient (Moran's I)*

#### Description

The Moran coefficient, a measure of spatial autocorrelation (also known as Global Moran's I)

#### Usage

 $mc(x, w, digits = 3, warn = TRUE, na.rm = FALSE)$ 

#### Arguments



# Details

The formula for the Moran coefficient (MC) is

$$
MC = \frac{n}{K} \frac{\sum_{i} \sum_{j} w_{ij} (y_i - \overline{y})(y_j - \overline{y})}{\sum_{i} (y_i - \overline{y})^2}
$$

where  $n$  is the number of observations and  $K$  is the sum of all values in the spatial connectivity matrix *W*, i.e., the sum of all row-sums:  $K = \sum_i \sum_j w_{ij}$ .

If any observations with no neighbors are found (i.e. any ( $Matrix::rowsums(w) == 0$ ) they will be dropped automatically and a message will print stating how many were dropped. The alternative is for those observations to have a spatial lage of zero—but zero is not a neutral value, see the Moran scatter plot.

<span id="page-7-0"></span>The Moran coefficient, a numeric value.

# Source

Chun, Yongwan, and Daniel A. Griffith. Spatial Statistics and Geostatistics: Theory and Applications for Geographic Information Science and Technology. Sage, 2013.

Cliff, Andrew David, and J. Keith Ord. Spatial processes: models & applications. Taylor & Francis, 1981.

#### See Also

[moran\\_plot,](#page-7-1) [lisa,](#page-0-0) [aple,](#page-0-0) [gr,](#page-0-0) [lg](#page-0-0)

# Examples

```
library(sf)
data(georgia)
w <- shape2mat(georgia, style = "W")
x <- georgia$ICE
mc(x, w)
```
<span id="page-7-1"></span>moran\_plot *Moran scatter plot*

#### Description

Plots a set of values against their spatially lagged values and gives the Moran coefficient as a measure of spatial autocorrelation.

#### Usage

```
moran_plot(
  x,
  w,
  xlab = "x (centered)",
 ylab = "Spatial Lag",
 pch = 20,
  col = "darkred",size = 2,
  alpha = 1,
  1wd = 0.5,
  na.rm = FALSE
)
```
# <span id="page-8-0"></span>moran\_plot 9

#### Arguments



# Details

For details on the symbol parameters see the documentation for [geom\\_point.](#page-0-0)

If any observations with no neighbors are found (i.e. any ( $Matrix::rowsums(w) == 0$ ) they will be dropped automatically and a message will print stating how many were dropped.

#### Value

Returns a gg plot, a scatter plot with x on the horizontal and its spatially lagged values on the vertical axis (i.e. a Moran scatter plot).

# Source

Anselin, Luc. "Local indicators of spatial association—LISA." Geographical analysis 27, no. 2 (1995): 93-115.

# See Also

[mc,](#page-6-1) [lisa,](#page-0-0) [aple](#page-0-0)

```
data(georgia)
x <- georgia$income
w <- shape2mat(georgia, "W")
moran_plot(x, w)
```
<span id="page-9-0"></span>

Draw samples from the posterior predictive distribution of a fitted geostan model.

#### Usage

```
posterior_predict(object, S, summary = FALSE, width = 0.95, car_parts, seed)
```
# Arguments



# Value

A matrix of size S x N containing samples from the posterior predictive distribution, where S is the number of samples drawn and N is the number of observations. If summary  $=$  TRUE, a data. frame with N rows and 3 columns is returned (with column names mu, lwr, and upr).

```
fit <- stan_glm(sents ~ offset(log(expected_sents)),
                 re = ~ m name,
                 data = sentencing,
                 family = poisson(),
                 chains = 2, iter = 600) # for speed only
yrep <- posterior_predict(fit, S = 65)
plot(density(yrep[1,]))
for (i in 2:nrow(yrep)) lines(density(yrep[i,]), col = 'gray30')
lines(density(sentencing$sents), col = 'darkred', lwd = 2)
```
<span id="page-10-0"></span>Simple features (sf) with historic (1910) county boundaries of Florida with aggregated state prison sentencing counts and census data. Sentencing and population counts are aggregates over the period 1905-1910, where populations were interpolated linearly between decennial censuses of 1900 and 1910.

#### Usage

sentencing

# Format

Simple features (sf)/data.frame with the following attributes:

name County name

wpop White population total for years 1905-1910

bpop Black population total for years 1905-1910

sents Number of state prison sentences, 1905-1910

plantation\_belt Binary indicator for inclusion in the plantation belt

pct\_ag\_1910 Percent of land area in agriculture, 1910

expected\_sents Expected sentences given demographic information and state level sentencing rates by race

sir\_raw Standardized incident ratio (observed/expected sentences)

#### Source

Donegan, Connor. "The Making of Florida's 'Criminal Class': Race, Modernity and the Convict Leasing Program." Florida Historical Quarterly 97.4 (2019): 408-434. <https://osf.io/2wj7s/>.

Mullen, Lincoln A. and Bratt, Jordon. "USABoundaries: Historical and Contemporary Boundaries of the United States of America," Journal of Open Source Software 3, no. 23 (2018): 314, [doi:10.21105/joss.00314.](https://doi.org/10.21105/joss.00314)

#### Examples

data(sentencing) print(sentencing)

<span id="page-11-1"></span><span id="page-11-0"></span>

Creates sparse matrix representations of spatial connectivity structures

# Usage

```
shape2mat(
 shape,
 style = c("B", "W"),
 queen,
 method = c("queen", "rook", "knn"),
 k = 1,longlat = NULL,
 snap = sqrt(.Machine$double.eps),
 t = 1,
 st.style = c("contemp", "lag"),
 quiet = FALSE
)
```
# Arguments



#### <span id="page-12-0"></span>shape2mat 13

#### Details

The method argument currently has three options. The queen contiguity condition defines neighbors as polygons that share at least one point with one another. The rook condition requires that they share a line or border with one another. K-nearest neighbors is based on distance between centroids. All methods are implemented using the spdep package and then converted to sparse matrix format.

Haining and Li (Ch. 4) provide a helpful discussion of spatial connectivity matrices (Ch. 4).

The space-time connectivity matrix can be used for eigenvector space-time filtering ([stan\\_esf](#page-13-1). The lagged' space-time structure connects each observation to its own past (one period lagged) value and the temporaneous' specification links each observation to its neighbors and to its own in situ past (one period lagged) value (Griffith 2012, p. 23).

# Value

A spatial connectivity matrix in sparse matrix format. Binary matrices are of class ngCMatrix, row-standardized are of class dgCMatrix, created by [sparseMatrix](#page-0-0).

#### Source

Bivand, Roger S. and Pebesma, Edzer and Gomez-Rubio, Virgilio (2013). Applied spatial data analysis with R, Second edition. Springer, NY. https://asdar-book.org/

Griffith, Daniel A. (2012). Space, time, and space-time eigenvector filter specifications that account for autocorrelation. Estadística Espanola, 54(177), 7-34.

Haining, Robert P. and Li, Guangquan (2020). Modelling Spatial and Spatial-Temporal Data: A Bayesian Approach. CRC Press.

#### See Also

[edges](#page-3-1) [row\\_standardize](#page-0-0) [n\\_nbs](#page-0-0)

```
data(georgia)
```

```
## binary adjacency matrix
C \leq - shape2mat(georgia, "B", method = 'rook')
```

```
## number of neighbors per observation
summary( n_nbs(C) )
head(Matrix::summary(C))
```

```
## row-standardized matrix
W <- shape2mat(georgia, "W", method = 'rook')
```

```
## summary of weights
E <- edges(W, unique_pairs_only = FALSE)
summary(E$weight)
```

```
## space-time matricies
## for eigenvector space-time filtering
```
14 stan\_esf

```
## if you have multiple years with same geometry/geography,
## provide the geometry (for a single year!) and number of years \code{t}
Cst <- shape2mat(georgia, t = 5)
dim(Cst)
EVst <- make_EV(Cst)
dim(EVst)
```
<span id="page-13-1"></span>stan\_esf *Spatial filtering*

### Description

Fit a spatial regression model using eigenvector spatial filtering (ESF).

# Usage

```
stan_esf(
  formula,
  slx,
  re,
  data,
 C,
 EV = make\_EV(C, nsa = nsa, threshold = threshold),nsa = FALSE,threshold = 0.25,
  family = gaussian(),
 prior = NULL,
 ME = NULL,centerx = FALSE,censor_point,
 prior_only = FALSE,
  chains = 4,
  iter = 2000,
  refresh = 500,keep\_all = FALSE,slim = FALSE,
  drop = NULL,
 pars = NULL,
  control = NULL,
  quiet = FALSE,...
)
```
# Arguments

formula A model formula, following the R [formula](#page-0-0) syntax. Binomial models are specified by setting the left hand side of the equation to a data frame of successes and failures, as in cbind(successes, failures)  $\sim$  x.

<span id="page-13-0"></span>

<span id="page-14-0"></span>

taining the model data. C Spatial connectivity matrix which will be used to calculate eigenvectors, if EV is not provided by the user. Typically, the binary connectivity matrix is best for calculating eigenvectors (i.e., using  $C =$  shape2mat(shape, style = "B")). This matrix will also be used to calculate residual spatial autocorrelation and any user specified slx terms; it will be row-standardized before calculating slx terms. See [shape2mat](#page-11-1).

 $alpha_re \sim N(0, alpha\_tau)$ alpha\_tau  $\sim$  Student\_t(d.f.,

- EV A matrix of eigenvectors from any (transformed) connectivity matrix, presumably spatial (see [make\\_EV](#page-5-1)). If EV is provided, still also provide a spatial weights matrix C for other purposes; threshold and nsa are ignored for user provided EV.
- nsa Include eigenvectors representing negative spatial autocorrelation? Defaults to nsa = FALSE. This is ignored if EV is provided.
- threshold Eigenvectors with standardized Moran coefficient values below this threshold value will be excluded from the candidate set of eigenvectors, EV. This defaults to threshold =  $0.25$ , and is ignored if EV is provided.
- family The likelihood function for the outcome variable. Current options are family = gaussian(), student\_t() and poisson(link = "log"), and binomial(link  $=$  "logit").
- prior A named list of parameters for prior distributions (see [priors](#page-0-0)):

intercept The intercept is assigned a Gaussian prior distribution (see [normal](#page-0-0).

beta Regression coefficients are assigned Gaussian prior distributions. Variables must follow their order of appearance in the model formula. Note that if you also use slx terms (spatially lagged covariates), and you use custom priors for beta, then you have to provide priors for the slx terms. Since slx terms are *prepended* to the design matrix, the prior for the slx term will be listed first.

sigma For family = gaussian() and family = student\_t() models, the scale parameter, sigma, is assigned a (half-) Student's t prior distribution. The half-Student's t prior for sigma is constrained to be positive.

nu nu is the degrees of freedom parameter in the Student's t likelihood (only used when family = student\_t()). nu is assigned a gamma prior distribution. The default prior is prior =  $list(nu = gamma2(alpha = 3, beta =$  $(0.2)$ .

<span id="page-15-0"></span>

#### <span id="page-16-0"></span>stan\_esf 17



#### Details

Eigenvector spatial filtering (ESF) is a method for spatial regression analysis. ESF is extensively covered in Griffith et al. (2019). This function implements the methodology introduced in Donegan et al. (2020), which uses Piironen and Vehtari's (2017) regularized horseshoe prior.

ESF decomposes spatial autocorrelation into a linear combination of various patterns, typically at different scales (such as local, regional, and global trends). By adding a spatial filter to a regression model, these spatial autocorrelation patterns are shifted from the residuals to the spatial filter. ESF models take the spectral decomposition of a transformed spatial connectivity matrix, C. The resulting eigenvectors,  $E$ , are mutually orthogonal and uncorrelated map patterns. The spatial filter equals  $E\beta_E$  where  $\beta_E$  is a vector of coefficients.

ESF decomposes the data into a global mean,  $\alpha$ , global patterns contributed by covariates  $X\beta$ , spatial trends  $E\beta_E$ , and residual variation. Thus, for family=gaussian(),

$$
y \sim Gauss(\alpha + X * \beta + E\beta_E, \sigma).
$$

An ESF component can be incorporated into the linear predictor of any generalized linear model. For example, a spatial Poisson model for rare disease incidence may be specified as follows:

$$
y \sim Poisson(e^{O+\mu})
$$
  
\n
$$
\mu = \alpha + E\beta_E + A
$$
  
\n
$$
A \sim Guass(0, \tau)
$$
  
\n
$$
\tau \sim student(20, 0, 2)
$$
  
\n
$$
\beta_E \sim horseshoe(.)
$$

The form of this model is similar to the BYM model (see [stan\\_icar\)](#page-19-1), in the sense that it contains a spatially structured trend term  $(E\beta_E)$  and an unstructured ('random effects') term (A).

<span id="page-17-0"></span>The [spatial.geostan\\_fit](#page-0-0) method will return  $E\beta_E$ .

The model can also be extended to the space-time domain; see [shape2mat](#page-11-1) to specify a space-time connectivity matrix.

The coefficients  $\beta_E$  are assigned the regularized horseshoe prior (Piironen and Vehtari, 2017), resulting in a relatively sparse model specification. In addition, numerous eigenvectors are automatically dropped because they represent trace amounts of spatial autocorrelation (this is controlled by the threshold argument). By default, stan\_esf will drop all eigenvectors representing negative spatial autocorrelation patterns. You can change this behavior using the nsa argument.

# Additional functionality:

The CAR models can also incorporate spatially-lagged covariates, measurement/sampling error in covariates (particularly when using small area survey estimates as covariates), missing outcome data, and censored outcomes (such as arise when a disease surveillance system suppresses data for privacy reasons). For details on these options, please see the Details section in the documentation for [stan\\_glm.](#page-0-0)

#### Value

An object of class class geostan\_fit (a list) containing:

summary Summaries of the main parameters of interest; a data frame

diagnostic Widely Applicable Information Criteria (WAIC) with a measure of effective number of parameters (eff\_pars) and mean log pointwise predictive density (lpd), and mean residual spatial autocorrelation as measured by the Moran coefficient.

data a data frame containing the model data

EV A matrix of eigenvectors created with w and geostan:: make\_EV

C The spatial weights matrix used to construct EV

family the user-provided or default family argument used to fit the model

formula The model formula provided by the user (not including ESF component)

- slx The slx formula
- re A list containing re, the random effects (varying intercepts) formula if provided, and data a data frame with columns id, the grouping variable, and idx, the index values assigned to each group.

priors Prior specifications.

- x\_center If covariates are centered internally (centerx = TRUE), then x\_center is a numeric vector of the values on which covariates were centered.
- ME The ME data list, if one was provided by the user for measurement error models.

spatial A data frame with the name of the spatial component parameter ("esf") and method ("ESF")

stanfit an object of class stanfit returned by rstan:: stan

# Author(s)

Connor Donegan, <connor.donegan@gmail.com>

stan\_esf 19

#### Source

Chun, Y., D. A. Griffith, M. Lee and P. Sinha (2016). Eigenvector selection with stepwise regression techniques to construct eigenvector spatial filters. *Journal of Geographical Systems*, 18(1), 67-85. [doi:10.1007/s1010901502253.](https://doi.org/10.1007/s10109-015-0225-3)

Dray, S., P. Legendre & P. R. Peres-Neto (2006). Spatial modelling: a comprehensive framework for principal coordinate analysis of neighbour matrices (PCNM). *Ecological Modeling*, 196(3-4), 483-493.

Donegan, C., Y. Chun and A. E. Hughes (2020). Bayesian estimation of spatial filters with Moran's Eigenvectors and hierarchical shrinkage priors. *Spatial Statistics*. [doi:10.1016/j.spasta.2020.100450](https://doi.org/10.1016/j.spasta.2020.100450) (open access: [doi:10.31219/osf.io/fah3z\)](https://doi.org/10.31219/osf.io/fah3z).

Donegan, Connor (2021). Building spatial conditional autoregressive (CAR) models in the Stan programming language. *OSF Preprints*. [doi:10.31219/osf.io/3ey65.](https://doi.org/10.31219/osf.io/3ey65)

Griffith, Daniel A., and P. R. Peres-Neto (2006). Spatial modeling in ecology: the flexibility of eigenfunction spatial analyses. *Ecology* 87(10), 2603-2613.

Griffith, D., and Y. Chun (2014). Spatial autocorrelation and spatial filtering, Handbook of Regional Science. Fischer, MM and Nijkamp, P. eds.

Griffith, D., Chun, Y. and Li, B. (2019). *Spatial Regression Analysis Using Eigenvector Spatial Filtering*. Elsevier.

Piironen, J and A. Vehtari (2017). Sparsity information and regularization in the horseshoe and other shrinkage priors. In *Electronic Journal of Statistics*, 11(2):5018-5051.

```
data(sentencing)
# spatial weights matrix with binary coding scheme
C <- shape2mat(sentencing, style = "B")
# log-expected number of sentences
## expected counts are based on county racial composition and mean sentencing rates
log_e <- log(sentencing$expected_sents)
# fit spatial Poisson model with ESF + unstructured 'random effects'
fit.esf <- stan_esf(sents ~ offset(log_e),
                   re = ~ m name,
                   family = poisson(),
                   data = sentencing,
                   C = C,
                   chains = 2, iter = 800) # for speed only
# spatial diagnostics
sp_diag(fit.esf, sentencing)
plot(fit.esf)
# plot marginal posterior distributions of beta_ev (eigenvector coefficients)
plot(fit.esf, pars = "beta_ev")
# plot the marginal posterior distributions of the spatial filter
plot(fit.esf, pars = "esf")
```

```
# calculate log-standardized incidence ratios
library(ggplot2)
library(sf)
f <- fitted(fit.esf, rates = FALSE)$mean
SSR <- f / sentencing$expected_sents
log.SSR < - log(SSR, base = 2)# map the log-SSRs
 ggplot(sentencing) +
   geom_sf(aes(fill = log.SSR)) +
   scale_fill_gradient2(
   midpoint = 0,
   name = NULL,
   breaks = seq(-3, 3, by = 0.5)) +
  labs(title = "Log-Standardized Sentencing Ratios",
    subtitle = "log( Fitted/Expected ), base 2"
 ) +
   theme_void()
```
<span id="page-19-1"></span>

stan\_icar *Intrinsic autoregressive models*

# Description

The intrinsic conditional auto-regressive (ICAR) model for spatial count data. Options include the BYM model, the BYM2 model, and a solo ICAR term.

# Usage

```
stan_icar(
  formula,
  slx,
  re,
  data,
  C,
  family = poisson(),
  type = c("icar", "bym", "bym2"),
  scale_factor = NULL,
 prior = NULL,
 ME = NULL,centerx = FALSE,
  censor_point,
 prior_only = FALSE,
  chains = 4,
  iter = 2000,
  refresh = 500,
```
<span id="page-19-0"></span>

<span id="page-20-0"></span>stan\_icar 21

```
keep_all = FALSE,
  slim = FALSE,
  drop = NULL,pars = NULL,control = NULL,quiet = FALSE,
  ...
\overline{)}
```
# Arguments



<span id="page-21-0"></span>

<span id="page-22-0"></span>

#### Details

The intrinsic conditional autoregressive (ICAR) model for spatial data was introduced by Besag et al. (1991). The Stan code for the ICAR component of the model and the BYM2 option is from Morris et al. (2019) with adjustments to enable non-binary weights and disconnected graph structures (see Freni-Sterrantino (2018) and Donegan (2021)).

The exact specification depends on the type argument.

#### ICAR:

For Poisson models for count data, y, the basic model specification (type = "icar") is:

```
y Poisson(e^{O+\mu+\phi})\phi \sim ICAR(\tau_s)\tau_s \sim Gauss(0,1)
```
where  $\mu$  contains an intercept and potentially covariates. The spatial trend *phi* has a mean of zero and a single scale parameter  $\tau_s$  (which user's will see printed as the parameter named spatial\_scale).

The ICAR prior model is a CAR model that has a spatial autocorrelation parameter  $\rho$  equal to 1 (see [stan\\_car\)](#page-0-0). Thus the ICAR prior places high probability on a very smooth spatially (or temporally) varying mean. This is rarely sufficient to model the amount of variation present in social and health data. For this reason, the BYM model is typically employed.

# BYM:

Often, an observational-level random effect term, theta, is added to capture (heterogeneous or unstructured) deviations from  $\mu + \phi$ . The combined term is referred to as a convolution term: convolution =  $\phi + \theta$ .

This is known as the BYM model (Besag et al. 1991), and can be specified using type = "bym":  $y \sim Poisson(e^{O+\mu+\phi+\theta})$ 

```
\phi \sim ICAR(\tau_s)\theta \sim Gaussian(0, \tau_{ns})\tau_s \sim Gaussian(0,1)\tau_{ns} \sim Gaussian(0,1)
```
The model is named after Besag, York, and Mollié (1991).

#### BYM2:

Riebler et al. (2016) introduce a variation on the BYM model (type = "bym2"). This specification combines  $\phi$  and  $\theta$  using a mixing parameter  $\rho$  that controls the proportion of the variation that is attributable to the spatially autocorrelated term  $\phi$  rather than the spatially unstructured term  $\theta$ . The terms share a single scale parameter  $\tau$ :

convolution = 
$$
[sqrt(\rho * S) * \tilde{\phi} + sqrt(1 - \rho)\tilde{\theta}] * \tau
$$
  
\n $\tilde{\phi} \sim Gaussian(0, 1)$   
\n $\tilde{\theta} \sim Gaussian(0, 1)$   
\n $\tau \sim Gaussian(0, 1)$ 

The terms  $\tilde{\phi}$ ,  $\tilde{\theta}$  are standard normal deviates,  $\rho$  is restricted to values between zero and one, and  $S$  is the 'scale\_factor' (a constant term provided by the user). By default, the 'scale\_factor' is equal to one, so that it does nothing. Riebler et al. (2016) argue that the interpretation or meaning of the scale of the ICAR model depends on the graph structure of the connectivity matrix C. This implies that the same prior distribution assigned to  $\tau_s$  will differ in its implications if C is changed; in other words, the priors are not transportable across models, and models that use the same nominal prior actually have different priors assigned to  $\tau_s$ .

Borrowing R code from Morris (2017) and following Freni-Sterrantino et al. (2018), the following R code can be used to create the 'scale factor'  $S$  for the BYM2 model (note, this requires the INLA R package), given a spatial adjacency matrix, C:

```
## create a list of data for stan_icar
icar.data <- geostan::prep_icar_data(C)
## calculate scale_factor for each of k connected group of nodes
k <- icar.data$k
scale_factor \leq vector(mode = "numeric", length = k)
for (j in 1:k) \{g.idx \leftarrow which(icar.data$comp_id == j)if (length(g.idx) == 1) {
       scale_factor[j] <- 1
       next
    }
```
<span id="page-24-0"></span>stan\_icar 25

```
Cg \leftarrow C[g.idx, g.idx]scale_factor[j] <- scale_c(Cg)
}
```
This code adjusts for 'islands' or areas with zero neighbors, and it also handles disconnected graph structures (see Donegan and Morris 2021). Following Freni-Sterrantino (2018), disconnected components of the graph structure are given their own intercept term; however, this value is added to  $\phi$  automatically inside the Stan model. Therefore, the user never needs to make any adjustments for this term. (To avoid complications from using a disconnected graph structure, you can apply a proper CAR model instead of the ICAR: [stan\\_car](#page-0-0)).

Note, the code above requires the scale\_c function; it has package dependencies that are not included in geostan. To use scale\_c, you have to load the following R function:

```
#' compute scaling factor for adjacency matrix, accounting for differences in spatial connectivity
#'
#' @param C connectivity matrix
#'
#' @details
#'
#' Requires the following packages:
#'
#' library(Matrix)
#' library(INLA);
#' library(spdep)
#' library(igraph)
#'
#' @source Morris (2017)
#'
scale_c \leq function(C) {
 geometric_mean <- function(x) exp(mean(log(x)))N = dim(C)[1]Q = Diagonal(N, rowSums(C)) - CQ pert = Q + Diagonal(N) * max(diag(Q)) * sqrt(.Machine$double.eps)
 Q_inv = inla.qinv(Q_pert, constr=list(A = matrix(1,1,N),e=0))scaling_factor <- geometric_mean(Matrix::diag(Q_inv))
 return(scaling_factor)
}
```
# Additional functionality:

The CAR models can also incorporate spatially-lagged covariates, measurement/sampling error in covariates (particularly when using small area survey estimates as covariates), missing outcome data, and censored outcomes (such as arise when a disease surveillance system suppresses data for privacy reasons). For details on these options, please see the Details section in the documentation for [stan\\_glm.](#page-0-0)

#### Value

An object of class class geostan\_fit (a list) containing:

summary Summaries of the main parameters of interest; a data frame

- diagnostic Widely Applicable Information Criteria (WAIC) with a measure of effective number of parameters (eff\_pars) and mean log pointwise predictive density (lpd), and mean residual spatial autocorrelation as measured by the Moran coefficient.
- stanfit an object of class stanfit returned by rstan:: stan
- data a data frame containing the model data
- edges The edge list representing all unique sets of neighbors and the weight attached to each pair (i.e., their corresponding element in the connectivity matrix C
- C Spatial connectivity matrix
- family the user-provided or default family argument used to fit the model
- formula The model formula provided by the user (not including ICAR component)
- slx The slx formula
- re A list with two name elements, formula and Data, containing the formula re and a data frame with columns id (the grouping variable) and idx (the index values assigned to each group).
- priors Prior specifications.
- x center If covariates are centered internally (centerx  $=$  TRUE), then x\_center is a numeric vector of the values on which covariates were centered.
- spatial A data frame with the name of the spatial parameter ("phi" if type  $=$  "icar" else "convolution") and method (toupper(type)).

#### Author(s)

Connor Donegan, <connor.donegan@gmail.com>

#### Source

Besag, J. (1974). Spatial interaction and the statistical analysis of lattice systems. *Journal of the Royal Statistical Society: Series B (Methodological)*, 36(2), 192-225.

- Besag, J., York, J., and Mollié, A. (1991). Bayesian image restoration, with two applications in spatial statistics. *Annals of the Institute of Statistical Mathematics*, 43(1), 1-20.
- Donegan, Connor and Morris, Mitzi (2021). Flexible functions for ICAR, BYM, and BYM2 models in Stan. Code repository. <https://github.com/ConnorDonegan/Stan-IAR>

Donegan, Connor (2021b). Building spatial conditional autoregressive (CAR) models in the Stan programming language. *OSF Preprints*. [doi:10.31219/osf.io/3ey65.](https://doi.org/10.31219/osf.io/3ey65)

Donegan, Connor and Chun, Yongwan and Griffith, Daniel A. (2021). Modeling community health with areal data: Bayesian inference with survey standard errors and spatial structure. *Int. J. Env. Res. and Public Health* 18 (13): 6856. DOI: 10.3390/ijerph18136856 Data and code: [https:](https://github.com/ConnorDonegan/survey-HBM) [//github.com/ConnorDonegan/survey-HBM](https://github.com/ConnorDonegan/survey-HBM).

Freni-Sterrantino, Anna, Massimo Ventrucci, and Håvard Rue (2018). A Note on Intrinsic Conditional Autoregressive Models for Disconnected Graphs. *Spatial and Spatio-Temporal Epidemiology*, 26: 25–34.

Morris, Mitzi (2017). Spatial Models in Stan: Intrinsic Auto-Regressive Models for Areal Data. [https://mc-stan.org/users/documentation/case-studies/icar\\_stan.html](https://mc-stan.org/users/documentation/case-studies/icar_stan.html)

<span id="page-26-0"></span>Morris, M., Wheeler-Martin, K., Simpson, D., Mooney, S. J., Gelman, A., & DiMaggio, C. (2019). Bayesian hierarchical spatial models: Implementing the Besag York Mollié model in stan. *Spatial and spatio-temporal epidemiology*, 31, 100301.

Riebler, A., Sorbye, S. H., Simpson, D., & Rue, H. (2016). An intuitive Bayesian spatial model for disease mapping that accounts for scaling. *Statistical Methods in Medical Research*, 25(4), 1145-1165.

#### See Also

[shape2mat,](#page-11-1) [stan\\_car,](#page-0-0) [stan\\_esf,](#page-13-1) [stan\\_glm,](#page-0-0) [prep\\_icar\\_data](#page-0-0)

```
data(sentencing)
C <- shape2mat(sentencing, "B")
log_e <- log(sentencing$expected_sents)
fit.bym <- stan_icar(sents ~ offset(log_e),
                     family = poisson(),
                     data = sentencing,
                     type = "bym",
                     C = C,
                     chains = 2, iter = 800) # for speed only
# spatial diagnostics
sp_diag(fit.bym, sentencing)
# check effective sample size and convergence
library(rstan)
rstan::stan_ess(fit.bym$stanfit)
rstan::stan_rhat(fit.bym$stanfit)
# calculate log-standardized incidence ratios
# (observed/exected case counts)
library(ggplot2)
library(sf)
f <- fitted(fit.bym, rates = FALSE)$mean
SSR <- f / sentencing$expected_sents
log.SSR < - log(SSR, base = 2)ggplot(sentencing) +
  geom_sf(aes(fill = log.SSR)) +
  scale_fill_gradient2(
  low = "navy",high = "darkred") +labs(title = "Log-standardized sentencing ratios",
       subtitle = "log( Fitted/Expected), base 2") +
  theme_void() +
  theme(
  legend.position = "bottom",
   legend.key.height = unit(0.35, "cm"),
```

```
legend.key.width = unit(1.5, "cm")\lambda
```
#### waic *Widely Applicable Information Criteria (WAIC)*

#### Description

Widely Application Information Criteria (WAIC) for model comparison

#### Usage

```
waic(fit, pointwise = FALSE, digits = 2)
```
#### Arguments



# Value

A vector of length 3 with WAIC, a rough measure of the effective number of parameters estimated by the model Eff\_pars, and log predictive density Lpd. If pointwise = TRUE, results are returned in a data.frame.

# Source

Watanabe, S. (2010). Asymptotic equivalence of Bayes cross validation and widely application information criterion in singular learning theory. Journal of Machine Learning Research 11, 3571- 3594.

```
data(georgia)
fit \le stan_glm(log(rate.male) \sim 1, data = georgia,
                chains = 2, iter = 800) # for speed only
waic(fit)
```
# <span id="page-28-0"></span>Index

∗ datasets sentencing, [11](#page-10-0) aple, *[8,](#page-7-0) [9](#page-8-0)* edges, [4,](#page-3-0) *[13](#page-12-0)* expected\_mc, [5](#page-4-0) formula, *[14](#page-13-0)*, *[21](#page-20-0)* geom\_point, *[9](#page-8-0)* geostan *(*geostan-package*)*, [2](#page-1-0) geostan-package, [2](#page-1-0) gr, *[8](#page-7-0)* lg, *[8](#page-7-0)* lisa, *[8,](#page-7-0) [9](#page-8-0)* make\_EV, [6,](#page-5-0) *[15](#page-14-0)* mc, *[6](#page-5-0)*, [7,](#page-6-0) *[9](#page-8-0)* moran\_plot, *[8](#page-7-0)*, [8](#page-7-0) n\_nbs, *[13](#page-12-0)* normal, *[15](#page-14-0)*, *[21](#page-20-0)* poly2nb, *[12](#page-11-0)* posterior\_predict, [10](#page-9-0) prep\_icar\_data, *[4](#page-3-0)*, *[27](#page-26-0)* prep\_me\_data, *[16](#page-15-0)*, *[22](#page-21-0)* priors, *[15,](#page-14-0) [16](#page-15-0)*, *[21](#page-20-0)* row\_standardize, *[13](#page-12-0)* sampling, *[17](#page-16-0)*, *[23](#page-22-0)* sentencing, [11](#page-10-0) set.seed, *[10](#page-9-0)* shape2mat, *[4](#page-3-0)*, *[6,](#page-5-0) [7](#page-6-0)*, [12,](#page-11-0) *[15](#page-14-0)*, *[18](#page-17-0)*, *[27](#page-26-0)* sparseMatrix, *[13](#page-12-0)* spatial.geostan\_fit, *[18](#page-17-0)* stan, *[17](#page-16-0)*, *[23](#page-22-0)* stan\_car, *[10](#page-9-0)*, *[23](#page-22-0)*, *[25](#page-24-0)*, *[27](#page-26-0)*

stan\_esf, *[6](#page-5-0)*, *[13](#page-12-0)*, [14,](#page-13-0) *[27](#page-26-0)* stan\_glm, *[18](#page-17-0)*, *[25](#page-24-0)*, *[27](#page-26-0)* stan\_icar, *[4](#page-3-0)*, *[17](#page-16-0)*, [20](#page-19-0)

waic, [28](#page-27-0)